"DYNAMIC BEHAVIOUR OF COUPLED SHEAR WALL STRUCTURES"

by

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SYNOPSIS

This paper concerns the computation of dynamic properties and response for three-dimensional shear wall structures whose modes of deformation include combinations of lateral displacement and torsion. Natural periods are computed for a typical twenty storey apartment building, including variations in wall thicknesses and floor plan geometry. The maximum dynamic stresses and displacements are also computed when the structure is subjected to a typical design earthquake response spectrum; these are compared with calculations made from the 1970 National Building Code of Canada.

NOMENCLATURE

$\{F(t)\}$	=	vector of dynamic forces
g	=	gravitational acceleration
$[I_\theta]$	=	diagonal matrix of floor level mass moments of inertia
$[K]$, $[KP]$, $[KT]$	=	stiffness matrices
$[M]$	=	diagonal matrix of floor level masses
M	=	no. of wall assemblies
N	=	no. of reference levels
P_i, P_{ik}	=	forces acting on walls
t	=	time
T_i, T_{ik}	=	torsional moments acting on walls
$\ddot{u}(t)$	=	earthquake acceleration
x	=	height above base
$\{y\}$,$\{y\}_k$	=	vectors of floor level deflections
z, z_k	=	distance of wall or load from reference axis
$\{\theta\}$, $\{\theta\}_k$	=	vectors of floor level rotations

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INTRODUCTION

Recent developments in high rise building construction indicate that extensive use is being made of shear walls, inter-connected by floor slabs, as the basic load carrying system. Efficient and economic design of such buildings must include the capability of determining the portion of load carried by each shear wall component, during both static and dynamic loading. This particular investigation is concerned with the dynamic behaviour characteristics of general three dimensional shear wall structures, characterized by inter-connected walls such that the resulting behaviour is a combination of lateral and torsional deformation.

A method for the static analysis for complete multi-storey shear wall buildings using small order matrices and incorporating the use of the continuous connection technique for the analysis of coupled shear wall assemblies has been presented elsewhere (3,4,9)(III). This method is incorporated into the analysis developed in this investigation in order to determine the lateral and torsional vibration characteristics of tall shear wall structures. Coupled and uncoupled modes of lateral and torsional vibration of a typical shear wall apartment building are examined for the alternate cases of including or neglecting shear wall interaction. Several geometrical and stiffness parameters are varied and the effect of these variations on the natural periods is examined. In addition, the modal superposition technique is used to determine the peak stresses and deformations when the structure is subjected to a typical earthquake, using a standard design earthquake response spectrum. The dynamically determined maxima are compared with each other and with statically determined design quantities, using the loads calculated from the earthquake provisions of the 1970 National Building Code of Canada (11).

STIFFNESS OF ENTIRE STRUCTURE

The building is considered to be comprised of M wall assemblies, inter-connected by floor slabs. The lateral loads are assumed to be applied to the structure at N reference levels, normally at floor levels, but can also be at multiples of floor levels for tall buildings. For a given position of the vertical reference axis, the resultant lateral force P_i applied to the building at floor level "i" is positioned at distance z from the co-ordinate axis, as shown in Figure 1. In this paper, it is assumed that all lateral loads act in one direction, e.g. the y direction in Figure 1. The resultant system applied at the reference axis at floor "i" therefore consists of a direct force P_i and a twisting moment $T_i = P_i z$. For equilibrium, the foregoing force and twisting moment must be carried by the M wall assemblies, such that

$$P_i = \sum_{k=1}^{M} P_{ik} \tag{1}$$

$$T_i = \sum_{k=1}^{M} P_{ik} z_k + \sum_{k=1}^{M} T_{ik} \tag{2}$$

III Numerals in parentheses refer to corresponding items in Bibliography.

where P_{ik} and T_{ik} are the lateral force and twisting moment respectively carried by wall assembly "k" at reference level "i", and located at a distance z_k from the vertical reference axis. It is assumed that any wall assembly "k" can sustain lateral loads and twisting moments, and that these are given by

$$\{P\}_k = [KP]_k \{y\}_k \qquad (3a)$$

$$\{T\}_k = [KT]_k \{\theta\}_k \qquad (3b)$$

where $\{P\}_k$ and $\{T\}_k$ are the vectors of lateral load and twisting moments at the N reference levels, $\{y\}_k$ and $\{\theta\}_k$ are the corresponding displacement and rotation vectors, and $[KP]_k$ and $[KT]_k$ are the corresponding stiffness matrices, all for wall assembly "k".

The objective of this particular stage of the analysis is to develop stiffness relationships between the forces and twisting moments at the reference axis (designated by the vectors $\{P\}$ and $\{T\}$) and the corresponding displacements and rotations of the reference axis (designated by the vectors $\{y\}$ and $\{\theta\}$). The compatibility relationship between the displacements of wall assembly "k" and those of the reference axis are given by

$$\{y\}_k = \{y\} + z_k \{\theta\} \qquad (4a)$$

$$\{\theta\}_k = \{\theta\} \qquad (4b)$$

Substituting the above into the assembly relationships given by Egs. 3 yield the following

$$\{P\}_k = [KP]_k \{y\} + z_k [KP]_k \{\theta\} \qquad (5a)$$

$$\{T\}_k = [KT]_k \{\theta\} \qquad (5b)$$

Expanding equilibrium equations (1) and (2) to all N reference levels and then substituting the above into these equilibrium equations yields

$$\left\{\begin{array}{c} P \\ \hline T \end{array}\right\} = \left[\begin{array}{c|c} K_{11} & K_{12} \\ \hline K_{21} & K_{22} \end{array}\right] \left\{\begin{array}{c} y \\ \hline \theta \end{array}\right\} \qquad (6)$$

in which

$$[K_{11}] = \sum_{k=1}^{M} [KP]_k$$

$$[K_{12}] = \sum_{k=1}^{M} z_k [KP]_k = [K_{21}]^T$$

$$[K_{22}] = \sum_{k=1}^{M} z_k^2 [KP]_k + \sum_{k=1}^{M} [KT]_k$$

From the above, it is clear that $[K_{12}]$ and $[K_{21}]$ will vanish if the structure is symmetrical and the reference axis is located at the axis of symmetry. In general, these sub-matrices will vanish if the reference axis is located at the shear centre.

Stiffness relationships of the form shown in Eq. 3 have been developed for assemblies of different kinds, and a "catalogue" of such relationships can be used to enable the overall structure stiffness relationship of Eq. 6

to be written for a structure containing a multiplicity of assemblies of different kinds. The assemblies used in this analysis are the plane wall, the planar coupled shear wall, and a double channel coupled box wall, whose stiffness properties are discussed in references (3),(8),(9), and (10). Stiffness relationship for general asymmetrical single walls and coupled walls, including the effects of torsional warping, are discussed in references (6) and (13).

DYNAMIC ANALYSIS

The equations of motion for the free vibration of a structure whose motion consists of combined bending and torsion may be written in the form

$$[M]\begin{Bmatrix}\ddot{y}\\ \ddot{\theta}\end{Bmatrix} + \begin{bmatrix}K_{11} & K_{12}\\ K_{21} & K_{22}\end{bmatrix}\begin{Bmatrix}y\\ \theta\end{Bmatrix} = 0 \qquad (7)$$

If the reference axis is the centre of mass at each floor level, then the mass matrix M is given by

$$[M] = \begin{bmatrix}[M] & 0\\ 0 & [I_\theta]\end{bmatrix} \qquad (8)$$

in which

$[M]$ = the diagonal matrix of masses at each floor level

and $[I_\theta]$ = the diagonal matrix of mass moments of inertia at each floor level

If the shear centre coincides with the centre of mass at each floor level, then sub-matrices $[K_{12}]$ and $[K_{21}]$ are both zero, and Eq. 7 reduces to two uncoupled sets of equations, one for lateral motion and the other for torsion.

In the general coupled case, the assumption of harmonic motion at a natural frequency ω, when substituted into Eq. 7, yields a characteristic equation satisfied by 2N different values of ω, each having a corresponding eigenvector or mode shape. As discussed with reference to coupled bending and torsion of thin-walled beams (5), these frequencies and mode shapes can often be classified as predominantly flexural or predominantly torsional in character. These predominantly flexural and predominantly torsional frequencies are, in certain situations, very nearly equal to the frequencies calculated by neglecting the coupling between bending and torsion.

If the coupling between bending and torsion is extremely weak, e.g. when the centre of mass and the shear centre are nearly coincidental, sub-matrices $[K_{12}]$ and $[K_{21}]$ in Eq. 7 can be neglected, thereby reducing the situation to that of an uncoupled system. In such a case, two separate sets of eigenvalues and eigenvectors result, one set each for bending and torsion.

When the structure is subjected to an earthquake ground motion, the equation of motion is similar to Eq. 7, except that the right hand side contains the force terms, which is of the form

$$\{F(t)\} = -\ddot{u}\begin{Bmatrix}m\\ 0\end{Bmatrix} \qquad (9)$$

in which u is the earthquake acceleration as a function of time and {m} is the column vector of floor masses. The analysis considered herein is the normal modal superposition technique, utilizing the response spectra for the particular earthquake, as described in detail in a number of references (1,2).

BASIC EXAMPLE STRUCTURE

The basic example structure is a 20 storey apartment building, whose floor plan is shown in Figure 2. Each storey has a height of 8.75 ft. The floor slabs are 8 in. thick and all walls are 6 in. thick except the end walls, which are 8 in. thick. The modulus of elasticity of concrete is assumed to be 3×10^6 p.s.i. and its unit weight is assumed to be 135 lb./ft.^{3}. The method presented in reference (12) is used to approximate the effective widths of the floor slabs acting as deep beams spanning between walls.

For the dynamic analysis, it is assumed that the structure is subjected to a design earthquake with a maximum acceleration of 0.06 g, and has characteristics given by Housner's average response spectrum (7). The damping is assumed to be equivalent to 5% of critical viscous damping. Maximum stresses and deformations for each of the cases is computed by combining all modes using the root mean square summation.

A comparison is made with a static analysis conducted according to the earthquake provisions of the 1970 National Building Code of Canada. With reference to these provisions, the structure is assumed to be in zone 3, having an importance factor of 1.3, and using the K factor for a box system (K = 1.33).

DYNAMIC PROPERTIES

In computing the dynamic properties for the basic example structure, two different mathematical models for structural stiffness are compared. The first of these includes the effect of connecting beams as interacting elements in determining the properties of combined planar walls and channel groupings; this model is designated as "with wall interaction". The second model is the converse of the above, in which the effect of the connecting members is not considered, and is designated as "no wall interaction".

The dynamic properties for both the "interaction" and "no interaction" models are computed assuming no coupling between bending and torsion; the significant mode shapes and periods are given in Figures 3 and 4. The properties for the coupled bending and torsion case are also computed for the model with wall interaction and are shown in Figure 5. Natural periods are also compared in Table 1 for all three combinations. From this Table it can be seen that the wall interaction has a significant effect on the natural periods, whereas the coupled and uncoupled periods are nearly identical. For this reason, further calculations are always based on the model with wall interaction.

The natural periods of the structure were also computed for several cases in which the torsional stiffness [KT] for the different assemblies were neglected. The resulting coupled and uncoupled natural periods are shown in Table 2. These results show that the torsional stiffness of the assemblies used in this structure have a very small overall effect, having minor effects on the fundamental period and imperceptible effects on the higher periods. Neglecting all torsional stiffnesses results in an 8% difference in the fundamental period, whereas neglect of only the planar wall assemblies results in only a 1% difference in the fundamental period.

The geometry was modified slightly in several stages in order to study the effect of different eccentricities on the natural periods of the structure. The five stages are as follows, with the eccentricity of line of shear centres relative to centre of mass noted:

I. Basic example structure, but with wall thicknesses of assemblies A and B increased to 1 ft. (ave. eccentricity = 2.2 ft.)

II. Basic example structure (ave. eccentricity = 2 ft.)

III. As in stage I, but no opening in assembly A. (ave. eccentricity = 1.6 ft.)

IV. Basic example structure, but with wall thickness of assembly E increased to 8 in. (ave. eccentricity = 1.3 ft.)

V. Basic example structure, but with assembly B removed (ave. eccentricity = 0.56 ft.)

The resulting variation of natural periods is shown in Table 3.

Stages I, II and IV are very near to each other, with a maximum difference of 4% in the fundamental natural period. In these stages, the only geometrical change was the thickness of the various assemblies. Stage III, even though producing a relatively small difference in eccentuity, shows rather large period changes in all modes. This is due to the fact that this stage is achieved by changing the basic stiffness of assembly A from an interesting coupled planar wall system to a single planar wall, which would have a much larger stiffness. This would clearly result in decreased periods, as indicated in Table 3. Stage V has been produced by eliminating an assembly, but this seems to have little effect on the periods (which are near to Stages I and II) although the eccentricity change is relatively large.

The natural periods for the basic example structure were also computed using differing numbers of reference levels in the computation. In addition to the normal reference level at each floor level, calculations were also made for totals of 10 and 5 reference levels. The resulting variation in periods is shown in Table 4. From this table it can be seen that 10 reference levels produces natural periods having exactly the same calculated values as for 20 reference levels. Some differences arise, particularly with the higher periods, when only 5 reference levels are used.

Several additional variations in calculated natural periods are shown in Table 5. These include the case of a thirty storey structure having the same floor plan and a twenty storey structure with the same floor plan but constructed of lightweight (110 lb./ft.^{3}) concrete. For the thirty storey structure the effective average eccentricity is reduced to 0.05 ft. Consequently one would expect the coupled and uncoupled periods to be very nearly identical, which is as indicated in Table 5. However, the periods for the thirty storey structure are only slightly higher than for the basic twenty storey structure, which indicates that the behaviour is not analagous to that of a single cantilever of varying length and the same stiffness. This would be expected due to the complex interaction of the various assemblies at all floor levels. The effect of the lightweight concrete is to produce a 10% reduction in fundamental period for a 20% reduction in weight. This is exactly analagous to the situation in a single mass oscillater, the period of which is proportional to the square root of the mass.

RESPONSE TO EARTHQUAKE MOTION

Maximum deflections and stresses in the component wall assemblies were computed using five mathematical models, namely

A. Dynamic, coupled bending and torsion, with wall interaction.

B. Dynamic, uncoupled bending and torsion, with wall interaction.

C. Dynamic, uncoupled bending only, with wall interaction.

D. Dynamic, uncoupled bending and torsion, no wall interaction.

E. Static, 1970 National Building Code of Canada, with wall interaction.

Typical maximum deflection curves are shown in Figure 6 for the end wall assembly and for one of the interior wall assemblies. In both cases, the maximum deflections computed by the static method are lower than those computed by any of the dynamic methods. Dynamic model D yields the largest deflections, which differ considerably from any of the other combinations. This would be anticipated since it is the only case in which wall interaction is neglected, and the neglect of wall interaction has already been shown to have a profound effect on the dynamic properties.

Deflections from model A would be expected to yield the best results. From the figures, it can be seen that model B provides a reasonable approximation of the maximum deflection. Comparing the curves for the end walls and for the interior walls shows that the torsional component has little influence on the deflection of the interior walls, but has a large effect on the deflection of the end walls.

Figure 8 shows the rotation curves for those models which include torsion, i.e. models A, B and D. Again models A and B are reasonably near each other, whereas model D shows much larger rotations. This is consistent with previous observations relative to the lateral deflections.

Figure 9 shows the maximum longitudinal stresses at the base of the structure for several typical wall assemblies (due only to lateral load). The stress behaviour is consistent with the deformations shown in the previous figure. Mathematical model D yields the largest stresses, whereas models A and B are reasonably similar. Statically computed stresses based on the National Building Code of Canada are similar to those computed from model A in those sections of the building near the centroid, but are considerably lower at the extremities.

CONCLUSIONS

The building studied in this investigation is not symmetrical but does have a very low eccentricity (centre of mass is only two ft. from the line of shear centres). However, the analysis has shown that even in this case torsion has a significant effect on the dynamic behaviour of the structure. This is due to the fact that the predominantly torsional and predominantly bending periods are very near to each other. It is recommended that the coupled periods be computed for all shear wall structures, even though they appear to be nearly symmetrical. If the fundamental periods are near to each other, then the design calculations should include both the effect of bending and torsion. However, in buildings with small eccentricities, the uncoupled periods and mode shapes can provide a good approximation which will reduce the required effort considerably.

Furthermore, it has been shown that the interaction between walls provided by the connecting beams is extremely important and cannot be neglected. This would be expected, since static investigations (3,9) have shown that this interaction changes the fundamental behaviour of the structure. The periods computed when neglecting this interaction are vastly different, so that the resulting modal maxima determined from the design response spectra can also be vastly different, depending upon the sensitivity of the response spectrum in that range of periods. In the particular example given here, this neglect of interaction would result in a conservative design; however, this will not always be the case, since it depends on the degree of interaction, the range of periods, and the nature of the design response spectrum in that range of periods.

The parameter variations have shown several significant features. Firstly, the effect of torsional stiffness of planar assemblies is negligible and even other assemblies have relatively minor effects on the natural periods. However, this is only true when the resisting assemblies are widely spaced so that the torsional resistance due to lateral distance from the shear centre is significant.

Changes in eccentricity have little effect on the natural periods if these are due to wall thickness changes, but have significant effect if due to changes in the basic structural action. Consequently, eccentricity is not a single parameter from which one can deduce the effect on the dynamic properties.

Results also show that fewer reference levels can safely be used to reduce computational effect without seriously affecting the validity of the results. The required number of reference levels depends upon the accuracy with which the strucutral deformations are to be represented; 10 levels is probably adequate for structure in the 10 to 50 storey range.

The comparison of the dynamically computed maximum displacements and stresses with those computed statically from the National Building Code indicates that the static values are approximately correct whenever torsion is not important, but cannot of course take into account magnification due to torsion. However, the static values are certainly not conservative, even at relatively low implied ductility factors, since the comparison is made with earthquake excitation into the dynamic model at the lowest value of peak acceleration within that particular zone. Consequently, it is recommended that, in order to ensure safety, that tall shear wall structures in Canadian Zone 3, or its equivalent elsewhere, be designed using the dynamic approach. If a reasonable estimate of the probable once in a hundred year peak acceleration is available, then this should be used as input rather than the zone boundary peak acceleration.

ACKNOWLEDGEMENT

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MODE NO.	TYPE	PERIOD, SECONDS		
		UNCOUPLED, NO WALL INTERACTION	UNCOUPLED, WITH WALL INTERACTION	COUPLED WITH WALL INTERACTION
1	Torsion	1.58	0.977	1.004
2	Bending	1.45	0.909	0.905
3	Torsion	0.266	0.215	0.224
4	Bending	0.232	0.198	0.209
5	Torsion	0.095	0.084	0.088
6	Bending	0.083	0.078	0.082

TABLE 1
COUPLED AND UNCOUPLED NATURAL PERIODS FOR EXAMPLE STRUCTURE, SECONDS

MODE NO. AND TYPE	PERIOD, SECONDS					
	TORSIONAL STIFFNESS INCLUDED IN ALL ASSEMBLIES		TORSONAL STIFFNESS INCLUDED ONLY IN ASSEMBLIES B AND E		NO TORSIONAL STIFFNESS INCLUDED IN ANY ASSEMBLIES	
	UNCOUPLED	COUPLED	UNCOUPLED	COUPLED	UNCOUPLED	COUPLED
1 (Torsion)	0.977	1.004	0.984	1.012	1.033	1.084
2 (Bending)	0.909	0.905	0.909	0.906	0.917	0.916
3 (Torsion)	0.215	0.224	0.215	0.224	0.218	0.225
4 (Bending)	0.198	0.209	0.198	0.209	0.198	0.208
5 (Torsion)	0.084	0.088	0.084	0.088	0.084	0.088
6 (Bending)	0.078	0.082	0.078	0.082	0.077	0.081

TABLE 2 EFFECT OF ASSEMBLY TORSIONAL STIFFNESSES ON NATURAL PERIODS OF EXAMPLE STRUCTURE

	PERIOD, SECONDS				
ECCENTRICITY, FT.	2.2	2.0	1.6	1.3	0.56
MODE NO. AND TYPE	STAGE I	STAGE II	STAGE III	STAGE IV	STAGE V
1. (Torsional) UNCOUPLED	1.000	0.977	0.897	0.960	0.988
COUPLED	1.030	1.004	0.914	0.978	1.012
2. (Bending) UNCOUPLED	0.916	0.909	0.781	0.911	0.913
COUPLED	0.906	0.905	0.759	0.909	0.907
3. (Torsional) UNCOUPLED	0.216	0.215	0.176	0.210	0.217
COUPLED	0.225	0.224	0.184	0.214	0.225
4. (Bending) UNCOUPLED	0.198	0.198	0.148	0.198	0.199
COUPLED	0.209	0.209	0.168	0.200	0.209
5. (Torsional) UNCOUPLED	0.084	0.084	0.067	0.082	0.085
COUPLED	0.087	0.088	0.069	0.084	0.089
6. (Bending) UNCOUPLED	0.077	0.078	0.052	0.078	0.078
COUPLED	0.082	0.082	0.063	0.079	0.082

STAGE I = Basic, plus A and B thickness increased to 1 ft.
STAGE II = Basic example structure
STAGE III = Stage I, plus no opening in A
STAGE IV = Basic, but with E thickness increased to 8 in.
STAGE V = Basic, but with B removed.

TABLE 3 EFFECT OF ECCENTRICITY CHANGE ON NATURAL PERIODS

MODE NO. AND TYPE	PERIOD, SECONDS					
	20 REFERENCE LEVELS		10 REFERENCE LEVELS		5 REFERENCE LEVELS	
	UNCOUPLED	COUPLED	UNCOUPLED	COUPLED	UNCOUPLED	COUPLED
1 (Torsion)	0.977	1.004	0.977	1.004	1.026	1.039
2 (Bending)	0.909	0.905	0.909	0.905	0.950	0.931
3 (Torsion)	0.215	0.224	0.215	0.224	0.226	0.233
4 (Bending)	0.198	0.209	0.198	0.209	0.207	0.216
5 (Torsion)	0.084	0.088	0.084	0.088	0.089	0.092
6 (Bending)	0.078	0.082	0.078	0.082	0.082	0.085

TABLE 4 EFFECT OF NO. OF REFERENCE LEVELS ON CALCULATED NATURAL PERIODS

MODE NO. AND TYPE	PERIOD, SECONDS					
	BASIC 20 STOREY EXAMPLE STRUCTURE		SAME FLOOR PLAN, BUT 30 STOREYS		SAME 20 STOREY STRUCTURE LIGHTWEIGHT CONCRETE	
	UNCOUPLED	COUPLED	UNCOUPLED	COUPLED	UNCOUPLED	COUPLED
1 (Torsion)	0.977	1.004	1.034	1.034	0.882	0.909
2 (Bending)	0.909	0.905	0.963	0.962	0.821	0.816
3 (Torsion)	0.215	0.224	0.239	0.240	0.194	0.207
4 (Bending)	0.198	0.209	0.219	0.219	0.179	0.185
5 (Torsion)	0.084	0.088	0.098	0.099	0.076	0.080
6 (Bending)	0.078	0.082	0.090	0.090	0.070	0.073

TABLE 5 NATURAL PERIODS FOR DIFFERENT STRUCTURES

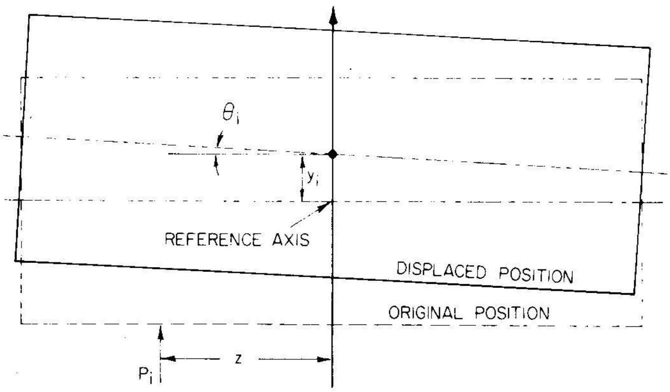

FIGURE. 1 MOVEMENT OF FLOOR "i" DUE TO FORCE P_i

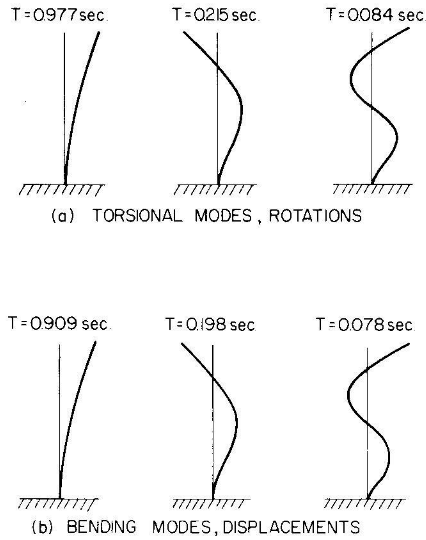

FIGURE. 3 MODE SHAPES AND PERIODS, UNCOUPLED MODES - WITH WALL INTERACTION

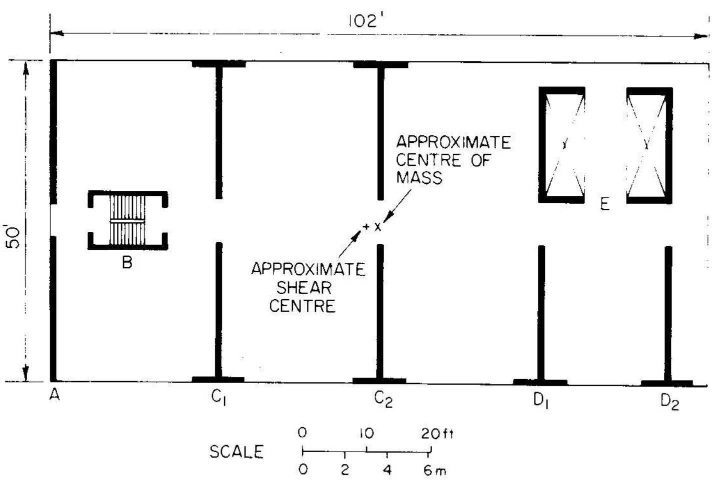

FIGURE. 2 FLOOR PLAN OF EXAMPLE STRUCTURE

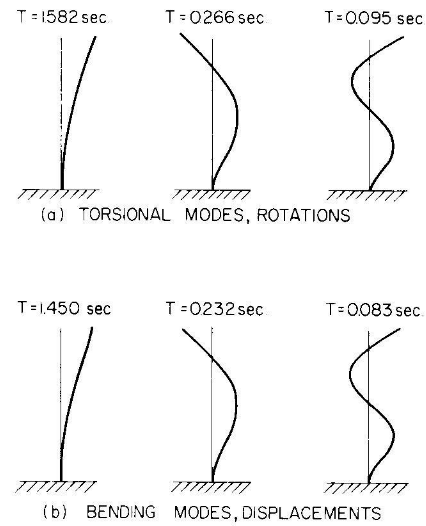

FIGURE. 4 MODE SHAPES AND PERIODS UNCOUPLED MODES, NO WALL INTERACTION

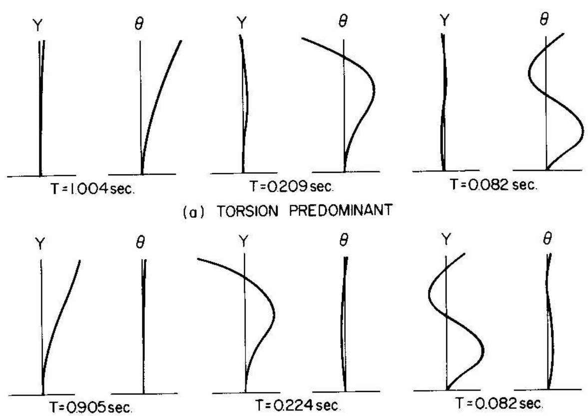

FIGURE. 5 MODE SHAPES AND PERIODS, COUPLED MODES WITH WALL INTERACTION

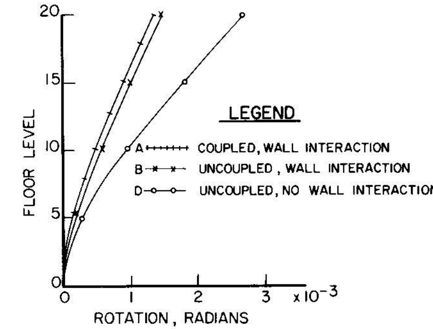

FIGURE. 7 MAXIMUM ROTATIONS OF STRUCTURE

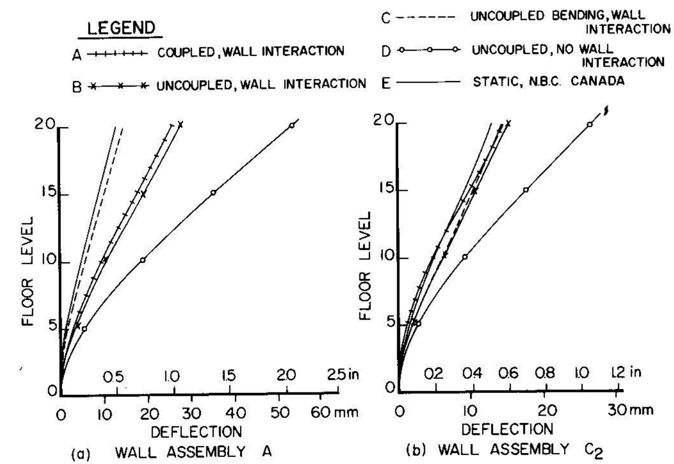

FIGURE. 6 MAXIMUM DEFLECTIONS OF TYPICAL WALL ASSEMBLIES

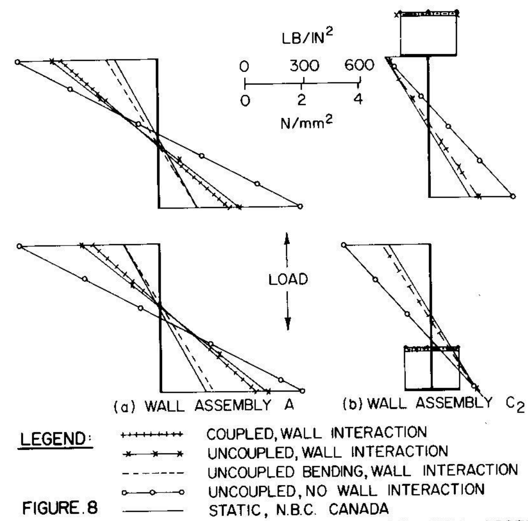

FIGURE. 8
MAXIMUM LONGITUDINAL STRESSES AT BASE, TYPICAL WALL ASSEMBLIES

DISCUSSION OF PAPER NO. 15

DYNAMIC BEHAVIOUR OF COUPLED SHEAR WALL STRUCTURES

by

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Written Discussion by: Dr. Ing. Riko Rosman, Faculty of Architecture, Zagreb, currently on leave at the University of Alberta, Edmonton

NOTE: The reader is directed to References 3 and 5 given at the end of Dr. Rosman's discussion for an explanation of the symbols used here. Some aspects of this discussion also refer to Paper No. 14.

The exact dynamic investigation of the simple perforated shear wall is highly involved. Thus, an approximate method of determining the most important dynamic property, the fundamental period of the free vibrations, of perforated shear walls and complete shear-wall building structures is highly desirable.[1 2 3]

In the following let us consider lateral vibrations of shear-wall structures symmetric in plan. Figure 1 shows some examples. If the simplifying assumptions

1. the mass of the structure is uniformly distributed along the structure height and

2. the vibration profile is similar to the lateral displacement profile due to a uniform lateral load

are made, the principle of conservation of energy may be applied to determine the vibration period:

$$T = 1.108 \sqrt{q \frac{\int_0^H \Delta^2 \, dx}{\int_0^H \Delta \, dx}} \qquad (1)$$

Notation:

- T vibration period [sec],
- q weight of the structure per unit height [kips/ft],
- Δ lateral displacement due to a unit uniform lateral load [ft].

Any other mass distribution along the structure height may be considered analogously, provided a correspondingly distributed lateral load is applied when determining the lateral displacement profile.

The displacements Δ are easily found using the continuous-connection concept.

Figure 2 shows some examples of shear-wall structures statically indeterminate to the first degree. The form of the lateral-displacement profile depends, in this case, only on two dimensionless quantities: on the stiffness parameter A of the structure and on the correction coefficient β which takes into account the effect of the extensional deformations of the piers adjacent to the connecting-beam bands. Figure 3 shows the form of the lateral-displacement profiles for some pairs of A and β.

Formula (1) yields

$$T = \eta \, . \, H^2\sqrt{\frac{q}{K}} \qquad (2)$$

where

- H structure height [m],
- K bending stiffness of the structure [Mpm^2], equal to the sum of the bending stiffnesses EI of all piers, sec/$\sqrt{m}$
- η period coefficient.

Figure 4 shows the diagram of the period coefficient η.

The lowest curve (β = 1) may also be applied for the approximate determination of the period highly redundant shear wall structures which are not too slender, so that the effect of the extensional deformation of the piers may be neglected.

Numerical investigations reveal, that with slender structures the effect of the extensional deformations of the piers may be considerable.

Concluding let it be remarked that both the formulae (1) and (2) and the period coefficient diagram (4) are valid not only for lateral vibrations but also for torsional vibrations of shear-wall structures symmetric in plan.
In this case, the weight q of the structure is to be replaced by the corresponding weight moment of inertia and the lateral stiffness parameters of the structure by the corresponding torsional stiffness parameters. [4 5]

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1. Rosman, R.: Statik und Dynamik der Scheibensysteme des Hochbaues. Springer Verlag Berlin Heidelberg New York, 1968.

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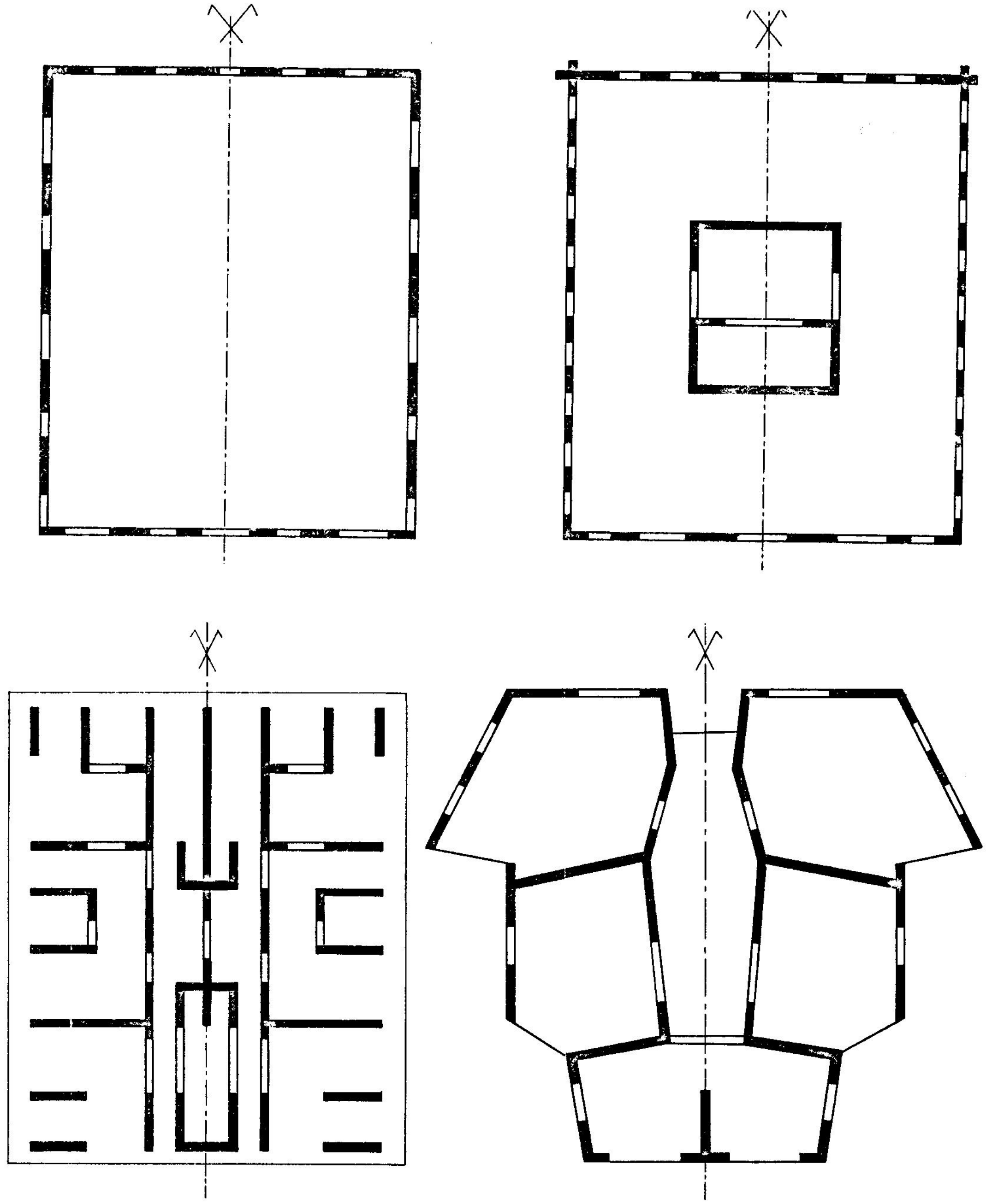

FIG. 1

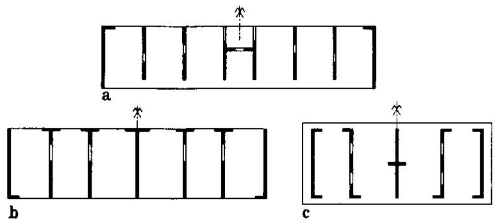

FIG. 2

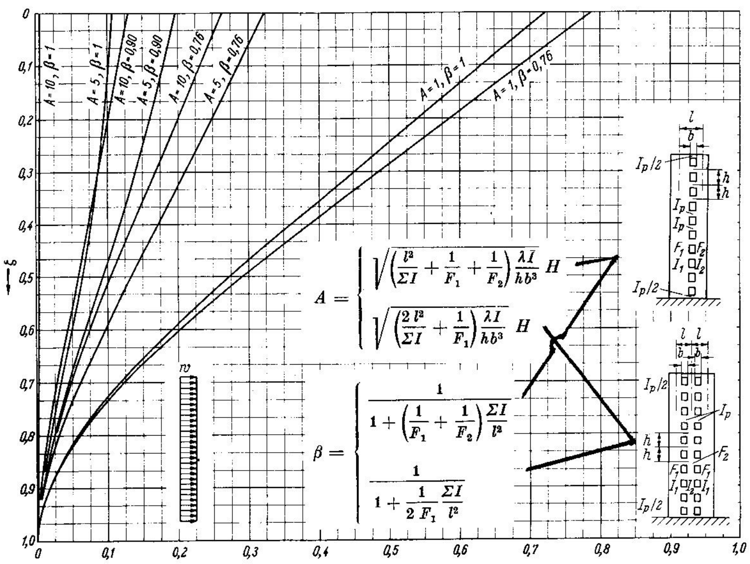

FIG. 3

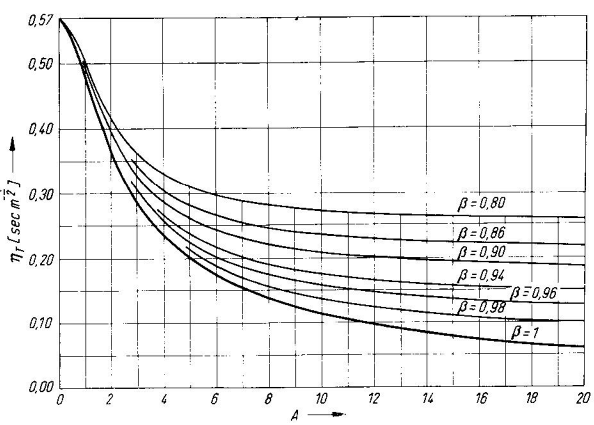

FIG. 4